Proper Platinum Resistance Thermometer Calibration Uncertainty Analysis
Introduction

• Uncertainty quantification is a primary task of the metrologist

• The uncertainty analysis must reflect our best understanding of the measurement and the instrument being calibrated

• If a calibration process is not fully understood, the uncertainty analysis is almost certainly flawed

• If the uncertainties do not reflect some aspects of UUT behavior, it will not reflect the UUT in use
Introduction

• Incomplete uncertainty analysis
  – Thorough uncertainty analysis is often more complex than it appears
  – Ignorance is bliss - the less we know, the better our numbers look
  – Although our assessors are very good, some do not possess sufficient current understanding to notice omissions in the uncertainty analysis

• Philosophical viewpoint
  – Should the calibration be considered as an isolated experiment or as part of a process?
  – Should the uncertainty analysis include only those components present at the time of calibration?
  – Should the uncertainty analysis include additional components to reflect UUT short term behavior?
  – Should the uncertainty analysis include additional components to reflect UUT long term behavior?
Introduction

• Consequently, it is easy to underestimate the real measurement uncertainty
  – Some omissions are insignificant to the final result and amount to a minor embarrassment or controversy when discovered
  – Other omissions may result in a noticeable, substantive underestimation of the final uncertainty
Categorization

- Often categorization helps us see things more clearly
- Platinum resistance thermometer calibration uncertainties can be categorized into two areas:
  - Uncertainties in temperature
  - Uncertainties in resistance measurement
Categorization

• Thermal uncertainties
  – Temperature stability
  – Temperature uniformity
  – Determination of the temperature
  – Temperature equilibrium
  – Self heating caused by excitation current*

• Resistance measurement uncertainties
  – Readout accuracy
  – Readout linearity
  – Electrical noise
  – Electrical interference
  – Self heating caused by excitation current*

*Evaluate only once (avoid double counting)
# Uncertainty Budget

## Uncertainty Evaluation

### Type A Evaluation

<table>
<thead>
<tr>
<th>Category</th>
<th>LN2 mK</th>
<th>tn100 mK</th>
<th>Hg mK</th>
<th>TPW mK</th>
<th>FPI nK</th>
<th>FPSn mK</th>
<th>FPZn mK</th>
<th>t500 mK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Readout Noise (1σ)</td>
<td>E</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>Total A</td>
<td></td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
</tr>
</tbody>
</table>

### Type B Evaluation

<table>
<thead>
<tr>
<th>Category</th>
<th>T</th>
<th>U</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPRT accuracy (calibration and drift)</td>
<td>1.50</td>
<td>1.91</td>
</tr>
<tr>
<td>Bath uniformity</td>
<td>1.15</td>
<td>1.92</td>
</tr>
<tr>
<td>Thermometer readout (6 ppm, SPRT)</td>
<td>0.20</td>
<td>0.03</td>
</tr>
<tr>
<td>Thermometer readout (6 ppm, UUT)</td>
<td>0.08</td>
<td>0.08</td>
</tr>
</tbody>
</table>

| Total B                        | 1.91    | 2.03    |
| Total Standard Uncertainty     | 1.92    | 2.04    |
| Total Expanded Uncertainty (k=2)| 3.84    | 4.08    |

**Total Standard Uncertainty (k=2):** 3.84 2.87 2.93 3.60 4.08 5.02 6.62
Additional Components

• Some components are missing
  – Reference SPRT $R_{TPW}$ propagation
  – Reference SPRT self heating correction
  – Reference SPRT immersion error
  – UUT noise contribution
  – UUT short term repeatability
  – UUT immersion error
  – UUT insulation resistance
  – Mathematical model uncertainties
  – Process repeatability
SPRT Uncertainties

• SPRT $R_{TPW}$ measurement (or ambiguity allowance) during use propagates to uncertainty in determination of $W_{T90}$
  - Example, $R_{TPW}$ uncertainty of 0.5 mK becomes 1.4 mK at 500 °C

• SPRT self heating is different in different thermal environments
  - Fixed point cells (SPRT cal) and comparison baths (use)

• SPRT has immersion requirements which may or may not be satisfied in the application
  - For example, calibration of short sensors
UUT Uncertainties

• UUT noise allowance
  – The noise influences the precision of the average obtained during the calibration

![Graph showing noise distribution in millikelvins (mK)]
UUT Uncertainties

• UUT short term repeatability
  – Hysteresis

![Hysteresis of 5 Industrial Probes](chart.png)
UUT Uncertainties

- UUT short term repeatability
  - $R_{TPW}$ repeatability (average = 100.0053, spread = 0.012 °C)
UUT Uncertainties

• UUT immersion error

Immersion Curves in a Liquid Bath at 80 °C
UUT Uncertainties

• UUT insulation resistance effects (worst case)

1) \[ R_{total}(\Omega) = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}} = \frac{1}{\frac{1}{256\Omega} + \frac{1}{10M\Omega}} \]

2) \[ R_{total}(\Omega) = 255.993447 \Omega \]

3) \[ \Delta R = 256\Omega - 255.993447 \Omega \approx 6.55 \times 10^{-3} \Omega \]

4) \[ \frac{6.55 \times 10^{-3} \Omega}{0.36 \Omega^0} = 1.82 \times 10^{-3} \approx 18.2 mK \]

5) \[ 18.2 mK \div \sqrt{3} = 10.5 mK \]
Mathematical Model U

Curve Fit

Magnitude of Residuals
Process Repeatability

Comparison Process Check Standard @ 420 °C
Standard Deviation = 0.00300 °C
## Revised Uncertainty Budget

### Uncertainty Evaluation

<table>
<thead>
<tr>
<th>Category</th>
<th>LN2 mK</th>
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<th>Hg mK</th>
<th>TPW mK</th>
<th>FPIn mK</th>
<th>FPSn mK</th>
<th>FPZn mK</th>
<th>t500 mK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process variability</td>
<td>P</td>
<td>0.90</td>
<td>1.80</td>
<td>1.30</td>
<td>1.00</td>
<td>1.99</td>
<td>2.60</td>
<td>3.20</td>
</tr>
<tr>
<td>Precision of UUT measurement</td>
<td>T&amp;E</td>
<td>2.22</td>
<td>1.67</td>
<td>1.67</td>
<td>1.11</td>
<td>1.67</td>
<td>1.67</td>
<td>2.22</td>
</tr>
<tr>
<td>Propagated repeatability of RTPW (UUT)</td>
<td>UUT</td>
<td>0.42</td>
<td>1.36</td>
<td>1.94</td>
<td>2.31</td>
<td>3.72</td>
<td>4.36</td>
<td>5.91</td>
</tr>
<tr>
<td><strong>Total A</strong></td>
<td>A</td>
<td>2.43</td>
<td>2.81</td>
<td>2.87</td>
<td>2.75</td>
<td>4.53</td>
<td>5.35</td>
<td>7.08</td>
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</tbody>
</table>

### Type B Evaluation

<table>
<thead>
<tr>
<th>Category</th>
<th>T</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPRT accuracy (calibration and drift)</td>
<td>1.50</td>
<td>1.50</td>
</tr>
<tr>
<td>SPRT RTPW propagation</td>
<td>0.05</td>
<td>0.17</td>
</tr>
<tr>
<td>SPRT self heating correction</td>
<td>0.58</td>
<td>0.58</td>
</tr>
<tr>
<td>Insulation resistance (UUT)</td>
<td>0.01</td>
<td>0.09</td>
</tr>
<tr>
<td>Bath uniformity</td>
<td>1.15</td>
<td>0.58</td>
</tr>
<tr>
<td>Thermometer readout (6 ppm, SPRT)</td>
<td>0.20</td>
<td>0.21</td>
</tr>
<tr>
<td>Thermometer readout (6 ppm, UUT)</td>
<td>0.03</td>
<td>0.08</td>
</tr>
<tr>
<td><strong>Total B</strong></td>
<td>B</td>
<td>1.99</td>
</tr>
<tr>
<td>Total Standard Uncertainty</td>
<td>U</td>
<td>3.15</td>
</tr>
<tr>
<td><strong>Total Expanded Uncertainty (k=2)</strong></td>
<td>U'</td>
<td>6.29</td>
</tr>
</tbody>
</table>

Thomas Wiandt

Fluke Corporation, Hart Scientific Division

NCSLI 2007 Session 8B
## Difference

<table>
<thead>
<tr>
<th></th>
<th>U'</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Original Uncertainty Evaluation</td>
<td>3.84</td>
<td>3.29</td>
<td>2.87</td>
<td>2.93</td>
<td>3.60</td>
<td>4.08</td>
<td>5.02</td>
</tr>
<tr>
<td>Revised Uncertainty Evaluation</td>
<td>6.29</td>
<td>6.61</td>
<td>6.54</td>
<td>6.38</td>
<td>9.99</td>
<td>11.73</td>
<td>15.71</td>
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<tr>
<td>Difference</td>
<td>Absolute</td>
<td>2.46</td>
<td>3.31</td>
<td>3.67</td>
<td>3.45</td>
<td>6.39</td>
<td>7.65</td>
</tr>
<tr>
<td></td>
<td>%</td>
<td>64.0</td>
<td>100.6</td>
<td>128.1</td>
<td>117.7</td>
<td>177.4</td>
<td>187.7</td>
</tr>
</tbody>
</table>
Conclusions

• Uncertainty evaluation involves variables that may not be readily apparent
• Some variables are difficult to quantify and may vary from UUT to UUT
• Underestimating the uncertainties in this manner may lead to significant errors
• Incomplete uncertainty analyses are very common among both unaccredited and accredited laboratories