

# Proper Platinum Resistance Thermometer Calibration Uncertainty Analysis

# Introduction

- Uncertainty quantification is a primary task of the metrologist
- The uncertainty analysis must reflect our best understanding of the measurement and the instrument being calibrated
- If a calibration process is not fully understood, the uncertainty analysis is almost certainly flawed
- If the uncertainties do not reflect some aspects of UUT behavior, it will not reflect the UUT in use

# Introduction

- Incomplete uncertainty analysis
  - Thorough uncertainty analysis is often more complex than it appears
  - Ignorance is bliss - the less we know, the better our numbers look
  - Although our assessors are very good, some do not possess sufficient *current* understanding to notice omissions in the uncertainty analysis

- Philosophical viewpoint
  - Should the calibration be considered as an isolated experiment or as part of a process?
  - Should the uncertainty analysis include only those components present at the time of calibration?
  - Should the uncertainty analysis include additional components to reflect UUT short term behavior?
  - Should the uncertainty analysis include additional components to reflect UUT long term behavior?

# Introduction

- Consequently, it is easy to underestimate the real measurement uncertainty
  - Some omissions are insignificant to the final result and amount to a minor embarrassment or controversy when discovered
  - Other omissions may result in a noticeable, substantive underestimation of the final uncertainty

# Categorization

- Often categorization helps us see things more clearly
- Platinum resistance thermometer calibration uncertainties can be categorized into two areas:
  - Uncertainties in temperature
  - Uncertainties in resistance measurement

# Categorization

- Thermal uncertainties
  - Temperature stability
  - Temperature uniformity
  - Determination of the temperature
  - Temperature equilibrium
  - Self heating caused by excitation current\*
- Resistance measurement uncertainties
  - Readout accuracy
  - Readout linearity
  - Electrical noise
  - Electrical interference
  - Self heating caused by excitation current\*

\*Evaluate only once (avoid double counting)

# Uncertainty Budget

## Uncertainty Evaluation

### Type A Evaluation

	Category	LN2 mK	tn100 mK	Hg mK	TPW mK	FPI <sub>n</sub> mK	FPS <sub>n</sub> mK	FPZ <sub>n</sub> mK	t500 mK
Readout Noise (1σ)	E	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20
<b>Total A</b>	<b>A</b>	<b>0.20</b>	<b>0.20</b>	<b>0.20</b>	<b>0.20</b>	<b>0.20</b>	<b>0.20</b>	<b>0.20</b>	<b>0.20</b>

### Type B Evaluation

SPRT accuracy (calibration and drift)	T	1.50	1.50	1.25	0.80	1.45	1.50	1.75	2.50
Bath uniformity	T	1.15	0.58	0.58	0.58	0.87	1.15	1.44	1.73
Thermometer readout (6 ppm, SPRT)	E	0.20	0.21	0.22	0.22	0.36	0.44	0.64	0.75
Thermometer readout (6 ppm, UUT)	E	0.03	0.08	0.11	0.84	0.24	0.29	0.42	0.54
<b>Total B</b>	<b>B</b>	<b>1.91</b>	<b>1.63</b>	<b>1.42</b>	<b>1.45</b>	<b>1.79</b>	<b>2.03</b>	<b>2.50</b>	<b>3.31</b>
Total Standard Uncertainty	U	1.92	1.65	1.43	1.46	1.80	2.04	2.51	3.31
<b>Total Expanded Uncertainty (k=2)</b>	<b>U'</b>	<b>3.84</b>	<b>3.29</b>	<b>2.87</b>	<b>2.93</b>	<b>3.60</b>	<b>4.08</b>	<b>5.02</b>	<b>6.62</b>

# Additional Components

- Some components are missing
  - Reference SPRT  $R_{TPW}$  propagation
  - Reference SPRT self heating correction
  - **Reference SPRT immersion error**
  - **UUT noise contribution**
  - UUT short term repeatability
  - UUT immersion error
  - UUT insulation resistance
  - **Mathematical model uncertainties**
  - Process repeatability



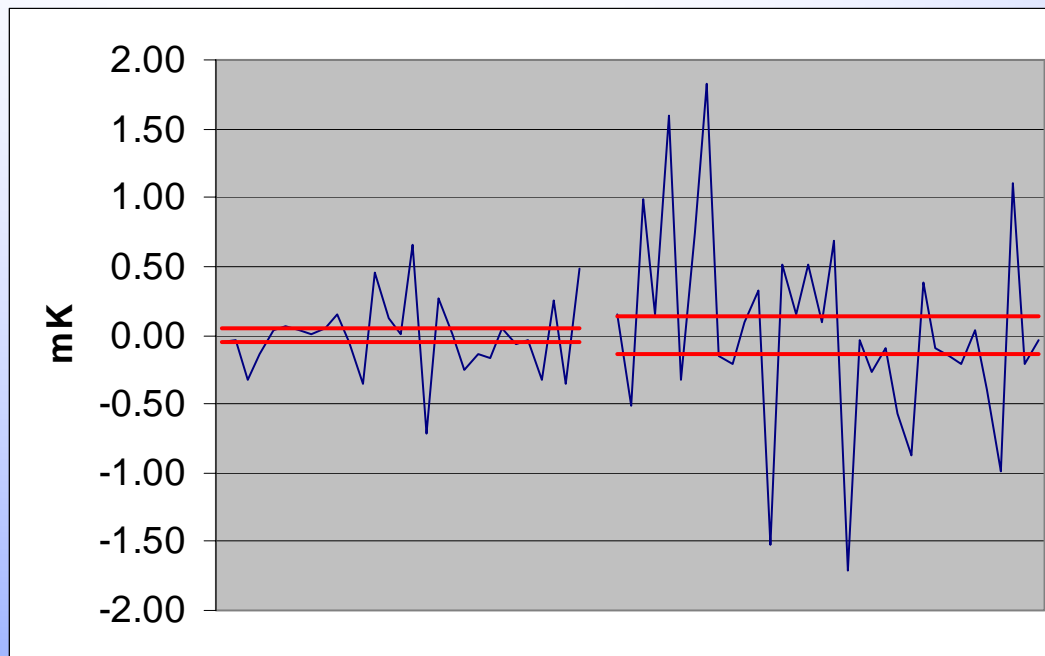
# SPRT Uncertainties

- SPRT  $R_{TPW}$  measurement (or ambiguity allowance) during use propagates to uncertainty in determination of  $W_{T90}$ 
  - Example,  $R_{TPW}$  uncertainty of 0.5 mK becomes 1.4 mK at 500 °C
- SPRT self heating is different in different thermal environments
  - Fixed point cells (SPRT cal) and comparison baths (use)
- SPRT has immersion requirements which may or may not be satisfied in the application
  - For example, calibration of short sensors

# UUT Uncertainties

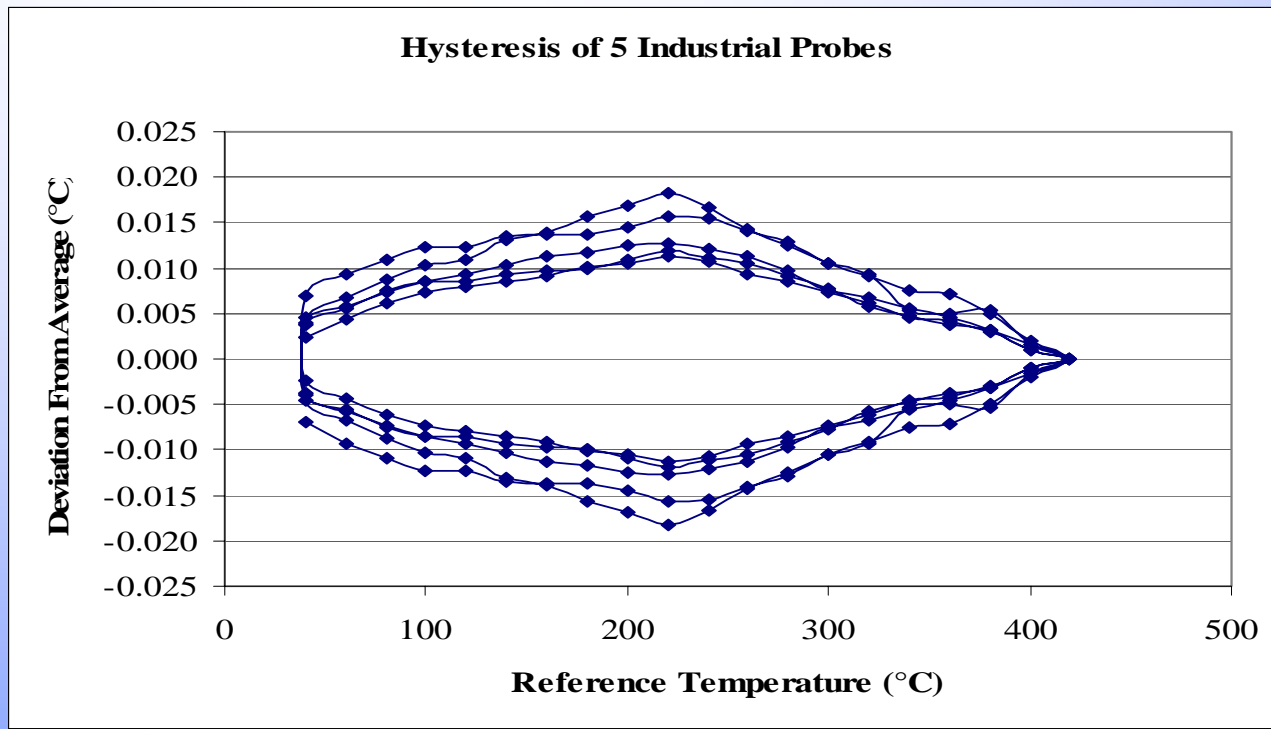
- UUT noise allowance

- The noise influences the precision of the average obtained during the calibration



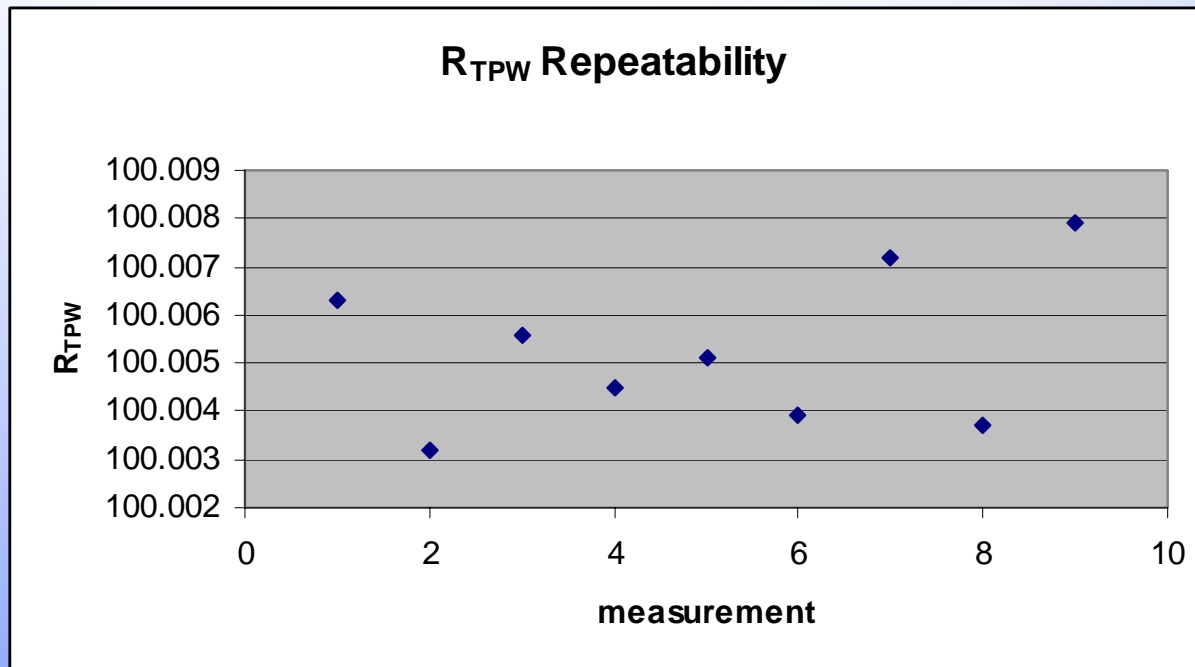
# UUT Uncertainties

- UUT short term repeatability
  - Hysteresis



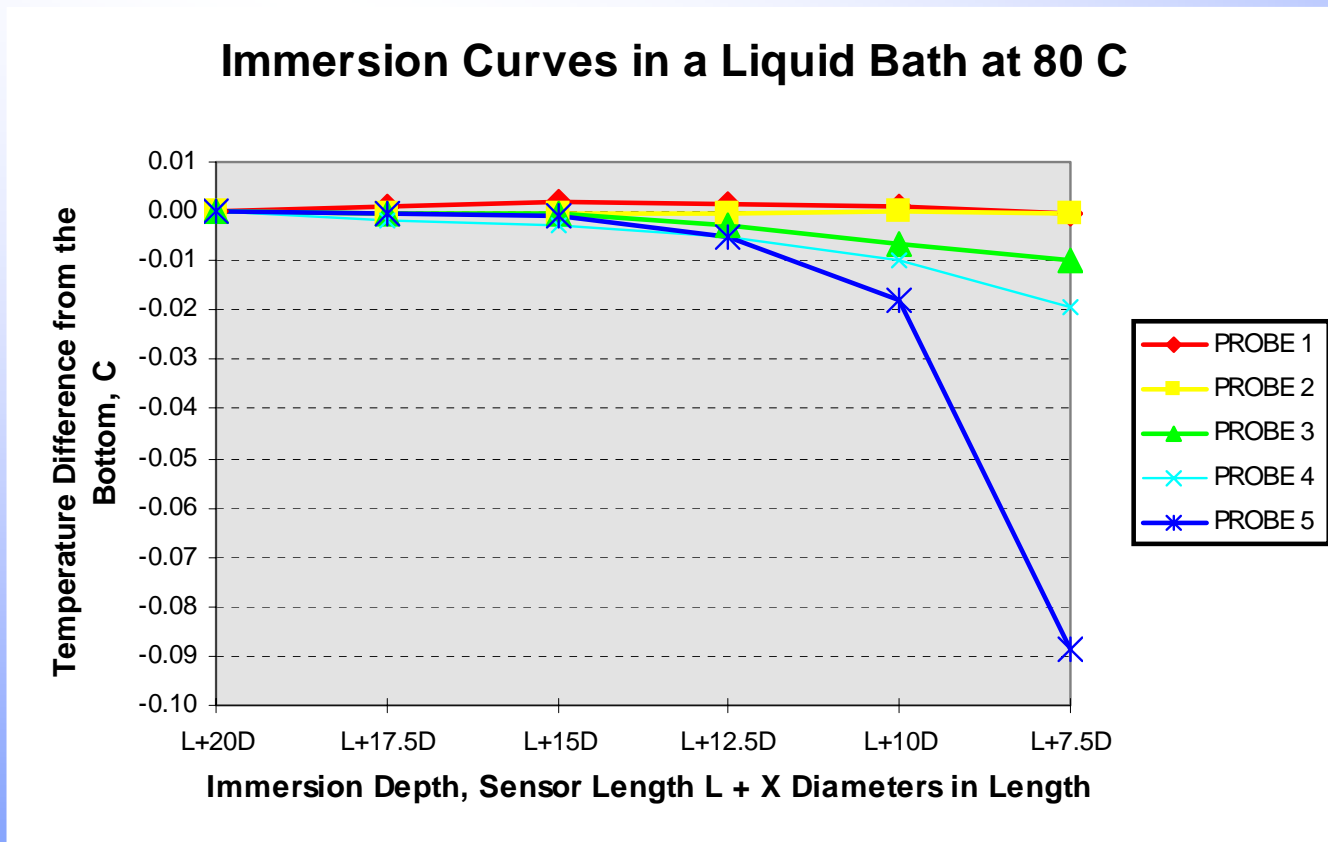
# UUT Uncertainties

- UUT short term repeatability
  - $R_{TPW}$  repeatability (average = 100.0053, spread = 0.012 °C)



# UUT Uncertainties

- UUT immersion error



# UUT Uncertainties

- UUT insulation resistance effects (worst case)

$$1) \quad R_{total}(\Omega) = \frac{1}{\frac{1}{R_1} + \frac{1}{R_1} + \dots + \frac{1}{R_n}} = \frac{1}{\frac{1}{256\Omega} + \frac{1}{10M\Omega}}$$

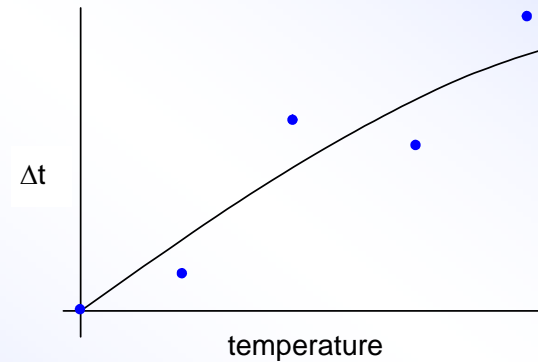
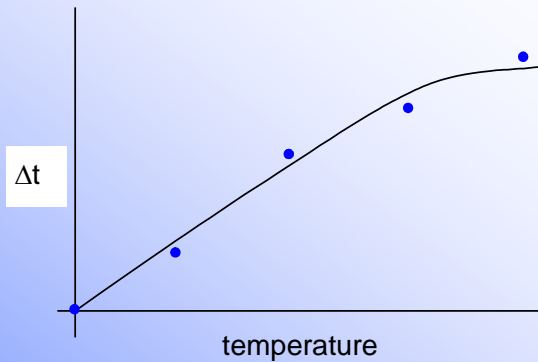
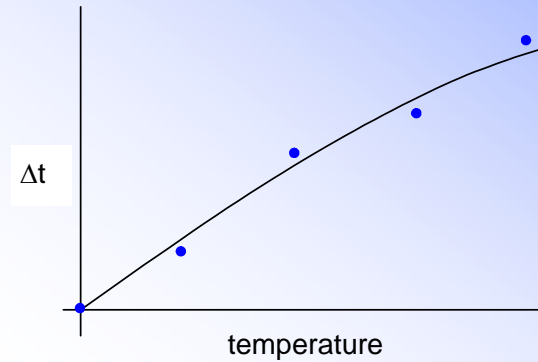
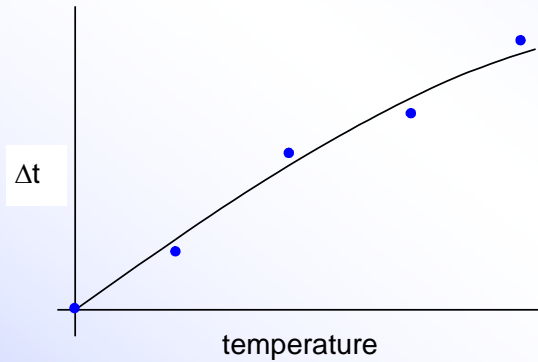
$$2) \quad R_{total}(\Omega) = 255.993447 \Omega$$

$$3) \quad \Delta R = 256\Omega - 255.993447 \Omega \approx 6.55 \times 10^{-3} \Omega$$

$$4) \quad 6.55 \times 10^{-3} \Omega / 0.36 \Omega/\%C = 1.82 \times 10^{-3} C = 18.2 mK$$

$$5) \quad 18.2 mK / \sqrt{3} = 10.5 mK$$

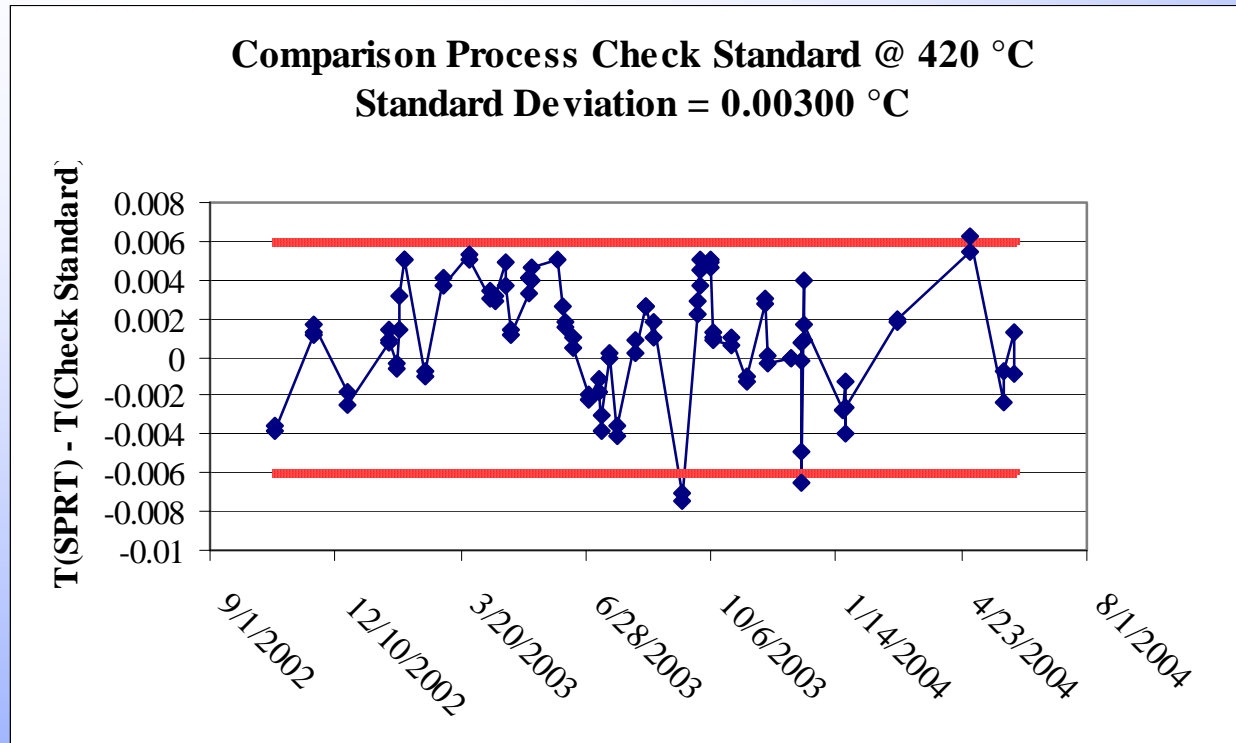
# Mathematical Model U



**Curve Fit**

**Magnitude of Residuals**

# Process Repeatability





# Revised Uncertainty Budget

## Uncertainty Evaluation

### Type A Evaluation

	Category	LN2 mK	tn100 mK	Hg mK	TPW mK	FPIIn mK	FPSn mK	FPZn mK	t500 mK
Process variability	P	0.90	1.80	1.30	1.00	1.99	2.60	3.20	3.70
Precision of UUT measurement	T&E	2.22	1.67	1.67	1.11	1.67	1.67	2.22	2.78
Propagated repeatability of R <sub>TPW</sub> (UUT)	UUT	0.42	1.36	1.94	2.31	3.72	4.36	5.91	6.47
<b>Total A</b>	<b>A</b>	<b>2.43</b>	<b>2.81</b>	<b>2.87</b>	<b>2.75</b>	<b>4.53</b>	<b>5.35</b>	<b>7.08</b>	<b>7.95</b>

### Type B Evaluation

SPRT accuracy (calibration and drift)	T	1.50	1.50	1.25	0.80	1.45	1.50	1.75	2.50
SPRT R <sub>TPW</sub> propagation	T	0.05	0.17	0.24	0.29	0.55	0.55	0.74	0.81
SPRT self heating correction	T	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58
Insulation resistance (UUT)		0.01	0.09	0.20	0.29	0.75	1.03	2.10	2.61
Bath uniformity	T	1.15	0.58	0.58	0.58	0.87	1.15	1.44	1.73
Thermometer readout (6 ppm, SPRT)	E	0.20	0.21	0.22	0.22	0.36	0.44	0.64	0.75
Thermometer readout (6 ppm, UUT)	E	0.03	0.08	0.11	0.84	0.24	0.29	0.42	0.54
<b>Total B</b>	<b>B</b>	<b>1.99</b>	<b>1.74</b>	<b>1.56</b>	<b>1.61</b>	<b>2.10</b>	<b>2.41</b>	<b>3.40</b>	<b>4.32</b>
Total Standard Uncertainty	U	3.15	3.30	3.27	3.19	5.00	5.86	7.85	9.05
<b>Total Expanded Uncertainty (k=2)</b>	<b>U'</b>	<b>6.29</b>	<b>6.61</b>	<b>6.54</b>	<b>6.38</b>	<b>9.99</b>	<b>11.73</b>	<b>15.71</b>	<b>18.10</b>

# Difference

<b>Original Uncertainty Evaluation</b>	<b>U'</b>	<b>3.84</b>	<b>3.29</b>	<b>2.87</b>	<b>2.93</b>	<b>3.60</b>	<b>4.08</b>	<b>5.02</b>	<b>6.62</b>
<b>Revised Uncertainty Evaluation</b>	<b>U'</b>	<b>6.29</b>	<b>6.61</b>	<b>6.54</b>	<b>6.38</b>	<b>9.99</b>	<b>11.73</b>	<b>15.71</b>	<b>18.10</b>
<b>Difference</b>	<b>Absolute</b>	<b>2.46</b>	<b>3.31</b>	<b>3.67</b>	<b>3.45</b>	<b>6.39</b>	<b>7.65</b>	<b>10.69</b>	<b>11.48</b>
	<b>%</b>	<b>64.0</b>	<b>100.6</b>	<b>128.1</b>	<b>117.7</b>	<b>177.4</b>	<b>187.7</b>	<b>213.1</b>	<b>173.3</b>

# Conclusions

- Uncertainty evaluation involves variables that may not be readily apparent
- Some variables are difficult to quantify and may vary from UUT to UUT
- Underestimating the uncertainties in this manner may lead to significant errors
- Incomplete uncertainty analyses are very common among both unaccredited and accredited laboratories