MATHEMATICAL CONSIDERATIONS BEHIND THE SWEEP SIZE-OF-SOURCE EFFECT METHOD

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Abstract: Size-of-source effect may be quantified in a number of different ways. These quantifications provide needed information for both the user of the radiation thermometer and for its calibration. For determination of size-of-source effect, there are a limited number of test methods furnished by published standards available today. The test methods available may be cumbersome. This paper discusses the mathematics behind an alternative method using a circular aperture and displacement from the center called the sweep method. It discusses this mathematical process in a step by step fashion.

1. INTRODUCTION

Knowledge of size-of-source effect (SSE) and field-of-view are essential knowledge for radiation thermometry use and calibration. However, testing for these two factors can be rather time consuming and cumbersome. Alternative methods have been suggested to test for size of source effect [1, 2, 3, 4, 5]. One such suggested method is the sweep method [6]. The sweep method is useful but has mathematical challenges that should be understood.

2. DESCRIPTION OF THE SWEEP METHOD

The traditional target size test method for determining SSE will be called the variable aperture method here [7, 8].

The sweep method is designed to test the same aspects of radiation thermometer performance that the variable aperture method achieves [6]. Instead of testing SSE by varying aperture size, SSE is tested by successive steps of displacing the radiation thermometer from center and observing the radiance. This is done by considering the field-of-view of the radiation thermometer as a finite number of shells. As the radiation thermometer is moved from center, certain shells are blocked and others are more exposed. An analysis of the sweep data and consideration of the exposure of each shell means that SSE can theoretically be determined from the sweep data.

To obtain SSE data from collected sweep data, a mathematical conversion from sweep data to SSE data is difficult. This is due to the sweep data being roughly a convolution of SSE data [6].

Due to this conversion being similar to a convolution, the following process is observed to obtain SSE data from sweep data. First, a guess of the radiation thermometer SSE is made. Then, the SSE data is converted to sweep data using the mathematical process described in the following sections. This calculated data is compared to the collected sweep data. Based on the differences, corrections to the SSE data are made. This is done in an iterative process until it converges to a solution.

3. WEIBULL DISTRIBUTION

It has been found that the Weibull cumulative distribution function (CDF) is a reasonable approximation for SSE [9, 10]. This is generally the case when 0.9 < SSE < 1.0 and of more interest when 0.99 < SSE < 1.00.

Two such cases are shown in Figure 1. The two cases shown in Figure 1 come from measured data using a variable aperture. In Figure 1, the measured data is represented by the points, and the Weibull fit is represented by the continuous curve. Figure 1a represents a radiation thermometer with a tightly focused field of view. Figure 1b represents a radiation thermometer with a more open field-of-view.

The idea behind using the Weibull CDF function goes back to the concept of combining an SSE pattern with perfect optics and the effects of optical scatter. The perfect optics forms a parabolic type shape and the scatter forms a curve similar to that of an exponential decay curve.

An example of such a fit is shown in Figure 2. A theoretical curve is constructed using a parabola in the region where SSE < 0.9 and an exponential decay curve in the region where SSE > 0.9. The theoretical data is represented by crosses (+). The fit using a Weibull CDF curve is represented by the curve.

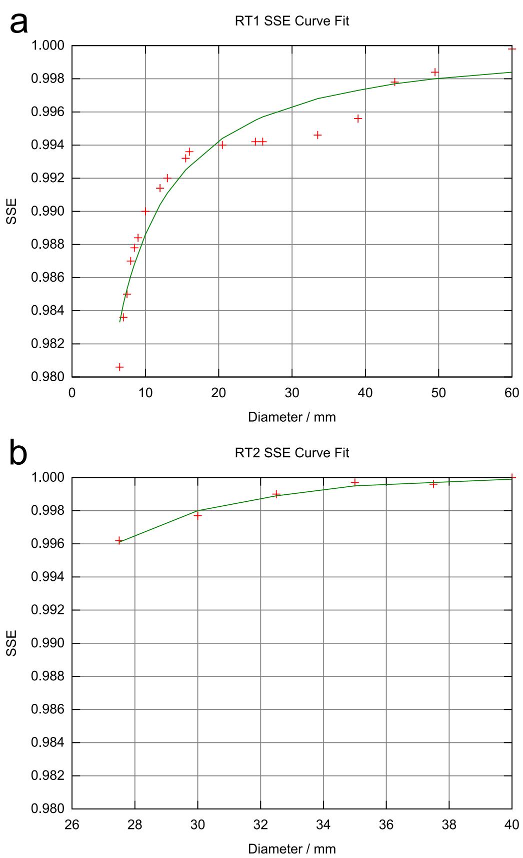

***Fig. 1.** Weibull PDF fitted to measured SSE data.*

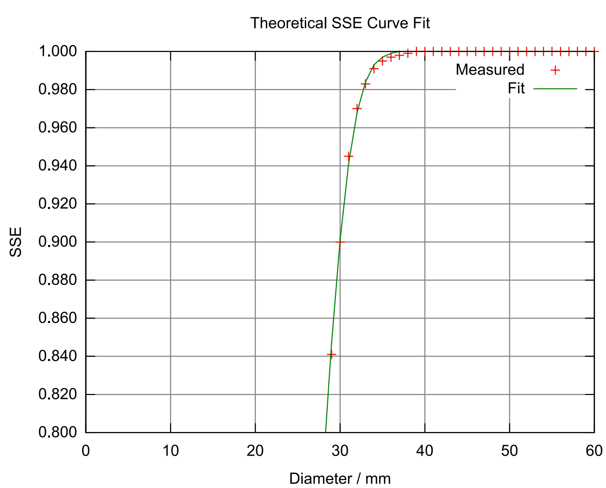

***Fig. 2.** Weibull fit accounting for ideal and non-ideal optical effects.*

The use of the Weibull CDF to fit size-of-source effect data is shown in Equation (1). The Weibull probability distribution function (PDF) is used to model radiance per area and becomes a power distribution function for SSE data. This is shown in Equation (2).

$$SSE(D) = 1 - \exp\left[-\left(\frac{D}{D_0}\right)^k\right] \qquad (1)$$

Where:
SSE(D) – size-of-source effect at diameter D
D – diameter of radiation thermometer field-of-view
D_0 – Weibull fitting parameter
k – Weibull fitting parameter

$$PDF(D) = \frac{k}{D_0}\left(\frac{D}{D_0}\right)^{k-1} \exp\left[-\left(\frac{D}{D_0}\right)^k\right] \qquad (2)$$

The conversion from the SSE to sweep data takes advantage of this. The SSE is modeled using a Weibull CDF. This CDF is run through the model described in the previous sections. Adjustments are then made to the Weibull CDF parameters to improve the fit to the sweep data. This is done in a few iterative steps.

4. SHELLS

The conversion of SSE data to sweep data involves breaking the SSE data into a finite number of ringed shaped shells [6]. This process is shown in Figure 3.

In Figure 3, each shell contains a portion of the radiance from the source and a portion of the radiance that comes from outside the source. In Figure 3, the radiance coming from the source is represented by the shaded area.

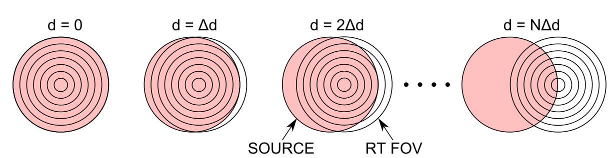

***Fig. 3.** Size-of-source effect shells and the sweep method.*

Each shell has a finite width bounded by two radii; the difference between the radii defines the shell width. Each radius will have a separate amount of radiance per unit area. Using a solid of revolution from integral calculus, the radiance for the shell can be calculated. This is represented by a CDF for the individual shell as is shown in Equation (3). Equation (4) shows the sum of all shells within a diameter (D).

$$CDF_{SHELL} = \frac{1}{3}\frac{2\left(h_2 r_2^2 - h_1 r_1^2\right) + \left(r_2 + r_1\right)\left(h_1 r_2 - h_2 r_1\right)}{r^2 - r^1} \quad (3)$$

r_1 – inside radius of shell
r_2 – outside radius of shell
h_1 – inside PDF of shell, PDF(r_1)
h_2 – outside PDF of shell, PDF(r_2)

$$SSE(D) = \sum_{i=1}^{N} CDF_{SHELLi} \quad (4)$$

5. DEALING WITH PARTIAL SHELLS

The numerical method depends on the calculation of how much of the SSE is contained within each shell. This computation is simple if the entire shell is contained within the aperture. It is also simple if none of the shell is contained within the aperture. However, if part of the shell is contained within the aperture and part of it outside, a consideration based on trigonometry must take place.

Partial shells are shown in Figure 3 as those where part of the shell is shaded and part is not.

Figure 4 shows a close up of a source represented by the white filled circle and a shell represented by the dark circle. The portion of the shell that is darkest is the portion of the shell containing radiance from the source.

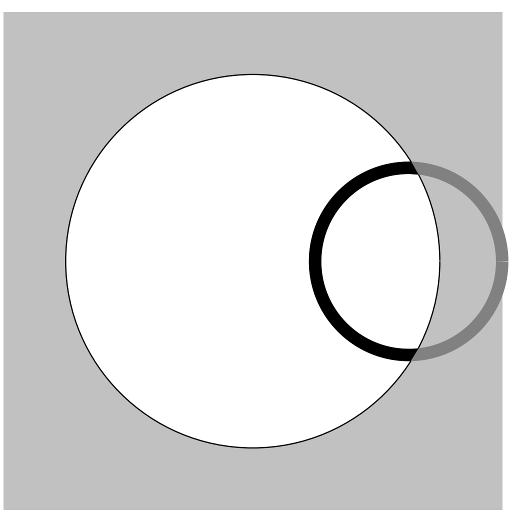

Fig. 4. *A partial shell.*

Figure 5 gives a diagram of the necessary parameters to calculate the amount of each shell contained within the source. In this figure, D_{APR} represents the source diameter, D_{SH} represents the shell diameter, and d represents the displacement of the radiation thermometer from the source's center. The parameters s_1 and s_2 facilitate the calculation for testing a shell's partiality. There are two relationships that they represent; $s_1 + s_2 = D_{APR}$ and $d + s_1 = D_{APR}/2$.

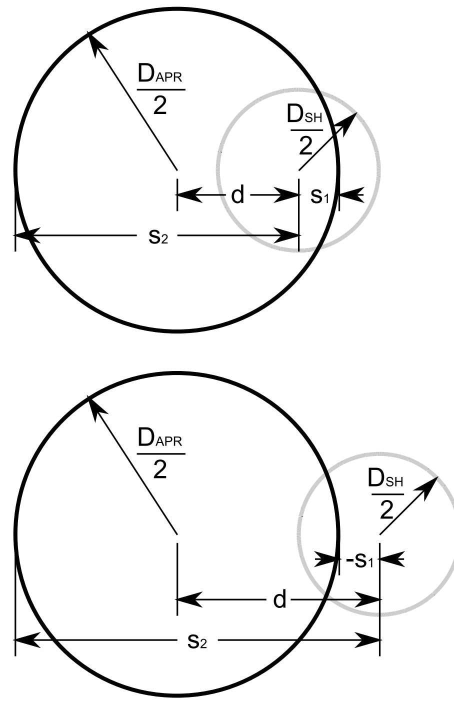

Fig. 5. *Testing for partial shells.*

A test must be done to determine if a shell is entirely inside the source (Type I), partially inside the source (Type II), or entirely outside of the source (Type III). A summary of this test is shown in Table 1. The fourth column of this table contains the logical test for each type. The parameters in the fourth column are those shown in Figure 5.

Type	Descip	Ratio of Shell	Test
I	Fully in	1	D_{SH} / 2 < s_1
II	Partial in	Eq. (5)	D_{SH} / 2 >= s_1 AND D_{SH} / 2 < s_2 OR D_{SH} / 2 > Abs(s_1) AND D_{SH} / 2 < s_2
III	Fully out	0	All other cases

Table 1. *Determination of type for shells.*

To calculate the amount or ratio of a shell that is within a source, the law of cosines is implemented. To do this, the shell-aperture geometry is analyzed as is shown in Figure 6. In this figure, the source is represented by the area within the solid circle. The shell is represented by the dashed line. The center of the radiation thermometer field-of-view is represented by the vertex of the triangle containing the angle Φ. To calculate the ratio, the angle Φ is divided by the angle of a semi-circle, π radians. This calculation is shown in Equation (5).

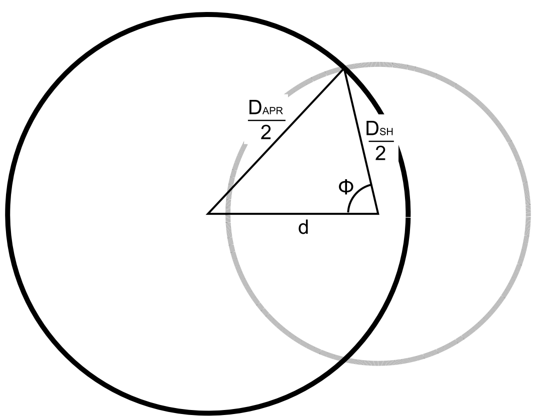

Fig. 6. *Geometry of partial shells.*

$$rat = \frac{1}{\pi}\arccos\left[\frac{\left(\frac{D_{SHELL}}{2}\right)^2 + d^2 - \left(\frac{D_{APR}}{2}\right)^2}{2\left(\frac{D_{SHELL}}{2}\right)d}\right] \quad (5)$$

Once the ratio is calculated for a given radius of field-of-view, the radiance from the source for the shell can be calculated. This is shown in Equation (6).

$$PDF_{PART}(r) = rat * PDF_{SHELL}(r) \quad (6)$$

The radiance PDF for both radii of the shell is used similar to Equation (3) to calculate the partial radiance of the shell as is shown in Equation (7).

$$CDF_{PARTi} = \frac{1}{3}\frac{2\left(h_2 r_2^2 - h_1 r_1^2\right) + \left(r_2 + r_1\right)\left(h_1 r_2 - h_2 r_1\right)}{r^2 - r^1} \quad (7)$$

The partial radiance for each shell is then summed to calculate the total radiance as shown in Equation (8).

$$SW(d) = \sum_{i=1}^{\infty} CDF_{PARTi} \quad (8)$$

Since there is some round off error, there is a procedure in place to minimize this error. It involves summing all of the shells in a model without an aperture and determining a correction factor based on how far the summation of SSE is from unity.

6. CONCLUSION

Using the SSE to sweep conversion as shown here is an alternate method to determine size-of-source effect for radiation thermometers. The mathematical process of this conversion has been outlined here. Some practical work has been done to verify this method, but more work needs to be done. The work shown here provides a good basis for others to do their own investigation.

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