Characterization of the Power Coefficient of AC and DC Current Shunts

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Abstract
It is common practice for current shunts to be calibrated at a single current level and then used over a wide range of currents. The power coefficient (PC) of the shunt can contribute significant error to the measurement process. Often the power coefficient is ignored either through lack of awareness or because it is specified to be acceptably small. For many shunts, however, the power coefficient is far from insignificant and an accurate characterization can provide a set of power coefficient corrections that can significantly reduce measurement uncertainty. This paper provides a summary of quite a few methods that can be used to measure or correct for power coefficient including a new method which the authors believe has not yet been published.

Introduction
Resistive shunts are often used in the measurement of current. For DC current, traceability is generally achieved through resistance and DC voltage. AC current is either achieved through the calibration of shunts for AC current or ac-dc difference of current. Usually it is assumed the power coefficient can be characterized with DC current and the same power coefficient can be used for AC current. However, some shunts are reported to have a bit of difference between their AC and DC power coefficients. Most of the methods described in this paper can be used for characterization of the power coefficient with either AC or DC current.

Method 1: Consider the Power Coefficient as Insignificant
This is the most common treatment of the power coefficient issue; either because the lab has quality shunts with low power coefficients compared to the overall uncertainty of measurement or because it was an error contributor that was overlooked with developing the uncertainty analysis. Too often, it is the latter. Current traceability is one of the parameters many laboratories do not get directly but derive from resistance and voltage. As a result, the uncertainty analysis can contain estimates of errors in current, resistance, and voltage which have to be converted through the use of sensitivity coefficients to current. It is easy to overlook some of the error contributors when they can occur in each area. Unfortunately power coefficient is an often overlooked error source.

If the metrologist determines to treat the power coefficient as insignificant, it is recommended that it be noted in the uncertainty analysis. That way it is clear when it is subsequently reviewed by an assessor or auditor it was intended to be treated as insignificant and was not overlooked.
Method 2: Include an Power Coefficient as Uncertainty Only

Often a laboratory will choose not to characterize its equipment for each of the influence factors. For example, the temperature coefficient of a resistor could be characterized and corrected for the current ambient temperature. Or the laboratory may decide to look at the range of temperatures expected and the temperature coefficient to assign an uncertainty for the resistance due to the temperature influence factor.

This approach may be taken for a shunt resistor. The manufacturer may publish a specification for power coefficient simply as a symmetric tolerance. Insufficient information is given to be able to make a correction for power coefficient even if the lab wanted to do so.

If the power coefficient is not specified, the lab may first have to use one of the methods listed in this paper to estimate the magnitude of the power coefficient for the uncertainty analysis. Even if a shunt has been characterized for power coefficient, the lab choose not to go to the effort to make corrections for power coefficient at each current level but to account for power coefficient with uncertainty.

Method 3: Obtain Power Coefficient as Part of a Calibration

The lab may find it easiest, though not always the most cost effective, to have the power coefficient characterized as part of the calibration. This could be reported on a certificate as either a power coefficient, a change resistance as a function of applied current, or as resistance values measured at a number of currents. Depending on the shunt design, it may be helpful to have the calibration provider attach a thermocouple to the shunt and record the shunt temperature at each current. It may be subsequently more accurate to correct shunt readings by making the appropriate corrections as a function of shunt temperature rather than as a function of shunt current.

Method 4: Characterize Power Coefficient with a Shunt of Known Power Coefficient.

If the laboratory has one shunt with a known or insignificant power coefficient, it can perform its own characterization of the power coefficient by comparing two shunts. Generally, power coefficients are characterized at DC and the resulting power coefficient is used for both AC and DC measurements. Though difference between the DC and AC power coefficients is usually insignificant, occasionally, for the most precise measurements, a characterization of both AC and DC power coefficients should be determined.

The simplest means of intercomparing shunts is to place them in series and measure the voltage across the reference standard (STD) and unit under test (UUT) standards. This configuration is shown in Figure 1. Care must be taken not to lose or inject current in the connections between the STD and UUT shunts, I_2 and I_3. These can be reduced by selecting meters with low bias current and low leakage currents. In addition, guarding can often help. Sometimes leakage currents are greater in the LO connection of meters than the HI connection. This configuration would result in I_1 being connected to LO and I_2 to HI of the meter reading the voltage across the UUT shunt. In addition, the GUARD of the UUT meter can be driven from the high the applied current. Any additional I_1 current flowing into the GUARD of the UUT meter will not cause any measurement error.
Figure 2 is a very similar configuration except that the STD is compared with a transfer (XFER) shunt and then the UUT shunt is substituted for the STD shunt. This will tend to cancel the errors due to $I_2$ and $I_3$.

![Figure 1. Method 4, direct comparison.](image1)

![Figure 2. Method 4, substitution.](image2)

If the current source is known to be stable and not sensitive to the compliance voltage changes if the shunt resistance are not equal values, then it may be possible to use the substitution method without needing to use the XFER shunt.

Some laboratories have been observed to compare two shunts using Method 4 and conclude that, if the shunts compare well at low and high currents, the shunts must have little power coefficient. This is a risky assumption, however. Many shunts use alloyed wire such as manganin which has a zero temperature coefficient (TC) infection point and a negative TC above and below. The authors have compared two shunts of the same model that compare to about 2-3 $\mu\Omega/\Omega$ between 10A and 20A. However, it has been found that both these shunts have about a 12 $\mu\Omega/\Omega$ shift in value between 10A and 20A. This situation is illustrated in Figure 3 below.

![Figure 3. Comparison of two shunts at 10A and 20A](image3)

**Method 5: Warm-up stabilization**

There are several methods that can be employed to characterize shunt power coefficients if the shunt has relatively large thermal mass and a relatively large thermal time constant such as the
One of the methods is to apply a stable current and then monitor the drift in shunt voltage using the configuration shown in Figure 5.

In this method, S1 is initially closed and the current source is set to the maximum shunt current to be characterized. The source is allowed plenty of time to stabilize. It is assumed that, when S1 is opened, the increase in compliance voltage will result in a negligible shift in the applied current and that the short term drift in the current source is also negligible. If these are not good assumptions, a second shunt can be used in series with the UUT to monitor the applied current. This shunt is warmed up and stabilized with the UUT shunt shorted. When the switch is opened, the readings of its voltage drop can be used to correct the applied current. It is not necessary to accurately know the exact value of the second shunt as the corrections should be fairly small. The resulting UUT waveform is shown in Figure 6 below.

Figure 4. High thermal time constant shunt.  

Figure 5. Warm-up stabilization.

Figure 6. DMM readings when S1 of Figure 4 is opened.
Figure 7 is the same chart of DMM readings as Figure 6 except with an expanded vertical scale. Sometimes because of the response time of the measuring system, the initial value must be estimated by projecting the response to Time = 0. A thermocouple may be attached to characterize the power coefficient as a function of temperature.

![Figure 7. DMM readings when S1 of Figure 4 is opened (expanded vertical scale).](image)

**Method 6: Cool-down stabilization**

Similar to Method 5, the cool-down stabilization method heats the shunt with full rated current with S1 closed. Then the applied current is dialed down to a lower level where the applied current can be accurately measured with the STD DMM and the voltage across the UUT measured with a second DMM. The configuration is shown in Figure 8 and the resulting UUT readings (corrected for applied current) are shown in Figure 9. As in the warm-up method, it may be necessary to project the shunt curve to Time = 0 to estimate the power coefficient. A thermocouple may also be attached to characterize the power coefficient as a function of temperature.

![Figure 8. Configuration for cool-down method.](image)
Method 7: Ovenized characterization with thermometer
Like Method 4, this method uses a direct comparison. However, only one current is measured relative to a STD shunt. The shunt resistance is calculated as a function of temperature. The thermometer is connected when the shunt is used to allow power coefficient corrections to be looked up and applied. Depending on the shunt construction, some differences in the UUT resistance can occur depending on whether the temperature rise is due to self-heating or externally heating in an oven [2].

Method 8: Binary buildup
Two shunts calibrated at lower current can be used to calibrate a higher current shunt. Two such calibrated shunts can become working standard shunts to calibrate at twice the original UUT
current. For high current shunts, the wiring is critical so the STD shunts will share somewhat equally. This method will work for AC only if the shunts and interconnecting are quite resistive at the frequencies being calibrated so the voltages across the STD shunts are in phase.

**Method 9: Buildup with two AC current sources**

In a paper [1] published by SP, the Swedish national laboratory, the configuration in Figure 12 was proposed for using two phase locked current sources of equal magnitude to provide a summed current for the calibration of the UUT shunt. If the current sources are accurate enough, the standard shunts would not be required.

The current sources are set to be equal in magnitude and are operated at this level during the calibration of the UUT shunt. By varying the phase from 180° to 0°, the current applied to the UUT shunt is continuously adjustable from 0 to twice the magnitude of the individual current sources.
Method 10: Use of a current transformer

Several metrologists from the Chinese national laboratory, NIM, published a method [3] using a binary inductive current divider to accurately form ratios of AC current of 10:1 or 100:1. They used this method to accurately determine the level dependence (power coefficient) of shunts for currents up to 100A and up to 100 kHz. This method is not described in detail here as it is complex enough that it is likely to be found at the national lab level but not in the toolbox of most working metrologists. Refer to the reference listed for a more detailed description.

Method 11: Segmented power coefficient characterization

The final method to be described in this paper has been used by the authors to characterize the power coefficient of a 100A shunt traceable through only a single 20A shunt. To the authors’ knowledge, this method has not been previously published.

The configuration is shown in Figure 13. Two current sources are employed. For the characterization of a 100A shunt described in this paper, the first current source is variable from 20A to 80A in 20A steps. This source needs to be reasonably stable but it is not necessary to know the value of the current extremely accurately.

A second current source needs to output 0A or 20A. It’s output is monitored with the STD shunt. Also, it is not necessary to know this shunt’s value extremely accurately.

First, the method will be illustrated using an idealized square law model, $PC = a \cdot I^2$. If the shunt has a 100 $\mu\Omega/\Omega$ power coefficient at 100A, $a = 0.01$. The illustration will use a single polarity for the 20A-80A source. To cancel some errors such as thermal EMF voltages, both polarities could be used and the results averaged. For the model, the UUT shunt is assumed to be 0.008$\Omega$, the STD shunt, 0.01$\Omega$, and the STAB shunt, 0.001$\Omega$.

![Figure 13. Segmented power coefficient characterization.](image-url)
<table>
<thead>
<tr>
<th>Current (A)</th>
<th>Power (W)</th>
<th>P coefficient (µΩ/Ω)</th>
<th>UUT DMM (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0.8</td>
<td>1</td>
<td>0.08000008</td>
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<td>20</td>
<td>3.2</td>
<td>4</td>
<td>0.16000064</td>
</tr>
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<td>7.2</td>
<td>9</td>
<td>0.24000216</td>
</tr>
<tr>
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<td>12.8</td>
<td>16</td>
<td>0.32000512</td>
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<td>100</td>
<td>80</td>
<td>100</td>
<td>0.80008000</td>
</tr>
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</table>

Table 1. Power and voltage “readings” for a 0.008 Ohm UUT shunt model.

Table 2 shows a portion of the spreadsheet that was employed to evaluate the validation model which was later used to enter the data when the power coefficient for a real shunt was characterized. The table is populated with the ideal values for the STAB and STD shunts and the “readings” calculated for the UUT DMM based on the power coefficient model for the UUT shunt. Note by using both polarities of current for the STD current source, duplicate points can be measured and averaged.

<table>
<thead>
<tr>
<th>20 to 80</th>
<th>0 to ±20</th>
<th>UUT</th>
<th>STAB DMM (mV)</th>
<th>STD DMM (mV)</th>
<th>UUT DMM (mV)</th>
<th>STD DMM (mV)</th>
</tr>
</thead>
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<tr>
<td>+20</td>
<td>+20</td>
<td>+40</td>
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<td>0.3200051</td>
<td>200.000000</td>
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<td>20.000000</td>
<td>-200.000000</td>
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<td>20.000000</td>
<td>0.000000</td>
<td>0.1600006</td>
<td>0.000000</td>
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<td>40.000000</td>
<td>200.000000</td>
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<td>200.000000</td>
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<td>+20</td>
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<td>-200.000000</td>
</tr>
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<td>0.000000</td>
<td>0.6400410</td>
<td>0.000000</td>
</tr>
</tbody>
</table>

Table 2. Power dissipated by a 0.008 Ohm shunt and shift in resistance for the model.

At each point, the STAB DMM and STD DMM are read twice, symmetrically in time about the UUT DMM readings. The symmetry of the readings tends to cancel drift in any of the shunts by averaging the DMM readings. Then using either nominal or known shunt values, the DMM readings are converted to current.

For each 20A transition, the change in the UUT DMM voltage is divided by the change in the applied current to calculate an incremental resistance for the UUT shunt as shown in Table 3.
For each incremental current an incremental voltage is produced. The sum of the incremental voltages is the total voltage and the sum of the incremental currents is the total current as described in Eq. 1 below. Figure 14 shows how these incremental values can be combined to produce the overall curve. One would expect, that to know the power coefficient accurately, the STAB and STD shunt values may need to be known accurately. However, as shown below, this method provides a means of greatly reducing the sensitivity of the power coefficient to the accuracy of the shunt values.

\[
V = V_1 + V_2 + \ldots + V_n = IR = I_1R_1 + I_2R_2 + \ldots + I_nR_n = IR_0(1 + \varepsilon) = I_1R_0(1 + \varepsilon_1) + I_2R_0(1 + \varepsilon_2) + \ldots + I_nR_0(1 + \varepsilon_n)
\]

Table 3. Incremental resistance calculated from the shunt “measurements” from Table 2

<table>
<thead>
<tr>
<th>Transition</th>
<th>Incremental Resistance</th>
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</thead>
<tbody>
<tr>
<td>20A to 40A</td>
<td>0.00800022</td>
</tr>
<tr>
<td>20A to 0A</td>
<td>0.00800003</td>
</tr>
<tr>
<td>40A to 60A</td>
<td>0.00800061</td>
</tr>
<tr>
<td>40A to 20A</td>
<td>0.00800022</td>
</tr>
<tr>
<td>60A to 80A</td>
<td>0.00800118</td>
</tr>
<tr>
<td>60A to 40A</td>
<td>0.00800061</td>
</tr>
<tr>
<td>80A to 100A</td>
<td>0.00800195</td>
</tr>
<tr>
<td>80A to 60A</td>
<td>0.00800118</td>
</tr>
</tbody>
</table>

Figure 14. Combination of resistance segments to build segmented curve
where: I is the total current and
\( \varepsilon \) is the power coefficient at current I
\( R_0 \) is the UUT shunt resistance value at low current; negligible power coefficient

Each segment can be expressed as:

- **Eq. 2**
  \[ V_j = I_j R_0 (1 + \varepsilon_j) \]
- **Eq. 3**
  \[ (1 + \varepsilon_j) = \frac{V_j}{I_j R_0} \]

At this point, this method can get into serious trouble. To get an accurate estimate of the power coefficient, the incremental voltage \( V_j \) across the UUT shunt, the voltage across the STD shunt, the value of the STD shunt, and \( R_0 \), the nominal (low current) UUT shunt value, must all be known accurately. The difficulty is that these errors will contribute to power coefficient error. If there is a 50 \( \mu \Omega/\Omega \) error in knowing \( R_0 \), an error of 50 \( \mu \Omega/\Omega \) will be propagated into calculation of the power coefficient; a 100 \( \mu \Omega/\Omega \) power coefficient would be calculated to be 150 \( \mu \Omega/\Omega \).

In the method the authors propose, incremental voltages and currents are measured and many of these error contributors cancel to a first degree. In this method, a 50 \( \mu \Omega/\Omega \) error in \( R_0 \) will only result in a 50 \( \mu \Omega/\Omega \) error in the value of the power coefficient. Thus a 100 \( \mu \Omega/\Omega \) power coefficient would be calculated to be 100.005 \( \mu \Omega/\Omega \). The main requirement is that the shunts and their measurements be stable and repeatable.

To achieve this, instead of using the measured or certified value of the shunt at low current to determine \( R_0 \), the first segment is used to calculate \( R_0 = \frac{V_1}{I_1} \). So, not only do we claim the shunt values do not have to be know very accurately but additional error is incurred by assuming the first segment has no power coefficient. The latter statement is only partially true. If, as is the case for most high current shunts, the first segment will have a relatively small but likely not negligible power coefficient. If we use five approximately equal current segments and if the power coefficient is proportional to the power dissipated in the shunt (proportional to the square of the applied current), the power at 20% of the current will be 4% of the power at full scale. Thus, after-the-fact we can apply 4% of the total power coefficient as the power coefficient for the first segment. This calculation could be iterated for greater accuracy but that is seldom necessary. The initial calculation can be used to test the hypothesis that the power coefficient is proportional to power. If it is not a linear function of power, the data can be fit to a curve before calculating how much power coefficient to apply to the first segment.

Now let us continue the calculations. Equation 3 can be approximated as:

- **Eq. 4**
  \[ (1 + \varepsilon_j) = \frac{V_j}{V_1/I_1} \]
\begin{equation}
\frac{V}{R_0} = \sum_{j=1}^{n} I_j + \sum_{j=1}^{n} I_j \left( \frac{V_j}{V_1} / \frac{I_j}{I_1} - 1 \right) = I + \sum_{j=1}^{n} I_j \left( \frac{V_j}{V_1} / \frac{I_j}{I_1} - 1 \right)
\end{equation}

\begin{equation}
\frac{V}{IR_0} = 1 + \sum_{j=1}^{n} \frac{I_j}{I} \left( \frac{V_j}{V_1} / \frac{I_j}{I_1} - 1 \right) = 1 + \sum_{j=1}^{n} \frac{I_j}{I} \varepsilon_j
\end{equation}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
n & Current (A) & Vn/ln / V1/I1 & \varepsilon & \varepsilon & \Sigma \varepsilon & \Sigma \varepsilon/n & \varepsilon_1 + \Sigma \varepsilon/n & Model \\
\hline
 & & & & & \mu \Omega/\Omega & \mu \Omega/\Omega & \mu \Omega/\Omega & \mu \Omega/\Omega \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 20 & 1.000000 & 0.000000 & 0 & 0 & 0 & 3.8 & 4 \\
2 & 40 & 1.000024 & 0.000024 & 24 & 24.0 & 12.0 & 15.8 & 16 \\
3 & 60 & 1.000072 & 0.000072 & 72 & 96.0 & 32.0 & 35.8 & 36 \\
4 & 80 & 1.00144 & 0.00144 & 144 & 240.0 & 60.0 & 63.8 & 64 \\
5 & 100 & 1.00240 & 0.00240 & 240 & 480.0 & 96.0 & 99.8 & 100 \\
\hline
\end{tabular}
\caption{Segment calculations for the shunt model}
\end{table}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{pc_curve.png}
\caption{Calculating PC curve for the shunt model}
\end{figure}

Table 4 shows the calculations for the shunt model and they are calculated in Figure 15. The power coefficient of the first segment was “sacrificed” to make the calibration less sensitive to the actual shunt values. Calculating that segment from the square law model and applying it results in only a 0.2 \( \mu \Omega/\Omega \) error as shown in the next to last column in Table 4.
This segmented approach was used by the authors to validate the power coefficient of a Fluke A40B-100A shunt. The equipment configuration is shown in Figure 15.

Figure 15. PC measurement system for the Fluke A40B.

Figure 16. Power coefficient results for the Fluke A40B.
Using the segmented method, it was determined the 100A power coefficient for the Fluke A40B tested was \(-37 \text{ } \mu \Omega/\Omega\). However, since this product is calibrated at 100A, no power coefficient need be applied for use at 100A. Power coefficient corrections or additional uncertainty needs to be applied for currents less than 100A.

This method should be able to be extended to AC current measurements as well. The two current sources would have to be phase locked. To add the currents, the phase would be set to approximately 0 and then adjusted to peak the reading on the UUT DMM. When subtracting the REF current, the phase would be set to approximately 180 and adjusted for a minimum (magnitude) UUT DMM reading.

**Conclusion**

Eleven methods of determining and dealing with shunt Resistive shunts have been presented including the segmented method. The metrologist is encouraged to become proficient at a number of these methods.

**References**

